Appearance of spatiotemporal noise and its effect on transport of electron pairs in a superconducting junction device

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Transport of electron pairs in a superconducting junction device with spatiotemporal noise is investigated. It is shown that, in a superconducting junction device, spatiotemporal noise can appear and induce a nonzero current due to the symmetry breaking induced by the correlation of the spatiotemporal noise with the phase difference. We found that there is a negative current for the electron pairs, exhibiting a well with increasing thermal noise strength. Our results can provide a theoretical foundation for the further investigation of superconducting junctions.

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I. INTRODUCTION

The net voltage (i.e., nonzero dc voltage with zero dc current) and the dc current-voltage characteristics in Josephson junctions (superconducting junctions) with noise has recently attracted increasing interest. Asymmetric noise can produce a net voltage [1,2] and dc voltage rectification [3,4]. We have shown that correlated symmetric noise can also produce a net voltage [2,5], which stems from a symmetry breaking of the system caused by the correlation between the additive and multiplicative noises. Zapata *et al.* investigated an asymmetric dc device with three Josephson junctions (depicted in Fig. 1) threaded by a magnetic flux and driven by a periodic signal and additive noise [6]. But they did not consider the case in the presence of additive and multiplicative noise, especially the case when the additive and multiplicative noises are correlated. We studied the net voltage, the dc current-voltage characteristics, and the mean first passage time for this device in the case of an environmental perturbation, together with thermal fluctuation [7]. (The environmental perturbation can be described by multiplicative noise in the Langevin equation [8–13], and the thermal fluctuation by additive thermal Gaussian white noise [14].) In Ref. [15], we investigated the transport of electron pairs for this superconducting junction device in the underdamped case. In our recent paper [16], the chaotic-noisy transport of electron pairs in this superconducting junctions device (thermal-inertia ratchets) was investigated. It was shown that, when the temperature is low enough, the transport of the electron pairs can be mainly chaotic; while when the temperature is high enough, it can be mainly stochastic. By controlling the temperature and the amplitude of the input ac signal, the current of electron pairs can be reversed. Savel'ev *et al.* investigated overdamped, directed transport that is controlled via the mixing of two periodic signals through different deterministic and Brownian ratchet setups [17]. The asymmetric dc device with three Josephson junctions described by Zapata *et al.* [6,18] is a suitable system to check all predictions made in Ref. [17]. Recently, Sterck *et al.* realized the device proposed by Zapata *et al.* [6,18] and demonstrated operation of those device as very efficient rocking ratchets [19].

However, all of the work for the superconducting junction device (or single super-conducting junction) has been focused only on the case of temporal noise (thermal Gaussian white noise or dichotomous noise; here the noise is only stochastic with respect to the time). In this paper, by analyzing the superconducting junction, we will introduce the appearance of spatiotemporal noise, which is stochastic with respect to the space and the time simultaneously, in the superconducting junction. Then we will investigate the effect of this noise on the transport of electron pairs in the Josephson junction device proposed by Zapata *et al.* (see Fig. 1).

II. APPEARANCE OF SPATIOTEMPORAL NOISE IN A SUPERCONDUCTING JUNCTION DEVICE AND OUR MODEL

The critical currents (i.e., J_1, J_2, and J_r) in Fig. 1 for the superconducting junction satisfy

$$J_i = \frac{e\hbar k_i}{m}, \tag{1}$$

in which $i=1,2$, and r, m is the mass of the electron pair, and k_i is the coupling between the insulating material and the superconductors. When an electron pair moves to the insu-

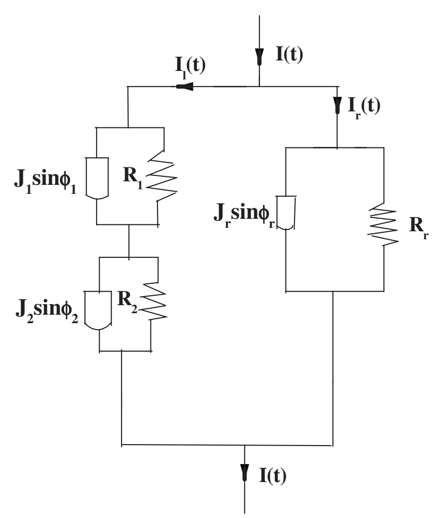

FIG. 1. Three-Josephson-junction device.

lating material from the superconductor, it has a coherence length λ_i in the insulating material. If the thickness L_i of the insulating material between the two superconductors is larger than λ_i, the electron pairs cannot tunnel through the insulating material between two superconductors, i.e., the coupling k_i is now zero. If the thickness L_i of the insulating material is smaller than λ_i, the electron pairs can tunnel through the insulating material between the two superconductors (i.e., the coupling k_i is now nonzero), and k_i is connected to the structure of the junction, such as the thickness L_i of the insulating material, the features of the superconducting material, and so on. Because k_i is connected with the thickness L_i of the insulating material between the two superconductors (note that now the electron pairs penetrate the insulating material for a depth L_i), and the phase difference between the two superconductors for the junction is related to L_i, k_i should be connected with the phase difference ϕ across the junction. But, for an experimental superconducting junction, we usually believe that k_i is a constant; this is because the connection between k_i and ϕ is very weak [20]. However, in this paper, as a physical theoretical investigation for the superconducting junctions device, we use

$$J_i = \frac{e\hbar k_i(\phi)}{m} = J_i(\phi), \tag{2}$$

where J_i is connected with ϕ very weakly.

In addition, if the variation of ϕ is not very large, we usually believe the resistance R_i ($i=1,2$, and r) in Fig. 1 is constant. But in practice R_i is generally correlated with the voltage $V(t)$ for the junctions [20,21]. Moreover, we know that $V(t)=(\hbar/2e)\dot{\phi}$ and $\dot{\phi}=-\omega_1\sin(\phi/2)-\omega_2\sin(\phi+\phi_e)$ [see Eq. (4) below] when $I(t)=0$. Thus, we have

$$R_i = R_i(V(t)) = R_i\left(\frac{\hbar}{2e}\dot{\phi}\right) = R_i(\phi), \tag{3}$$

in which the effect of the phase difference ϕ on the resistance R_i in Fig. 1 is very small.

Substituting Eqs. (2) and (3) into Eq. (3) of Ref. [7] (see Ref. [22]), we get [22]

$$\dot{\phi} = -\omega_1(\phi)\sin\frac{\phi}{2} - \omega_2(\phi)\sin(\phi+\phi_e) + \frac{eR(\phi)}{\hbar}I(t), \tag{4}$$

in which $\omega_1=[eR(\phi)/\hbar]J_l(\phi)$ and $\omega_2=[eR(\phi)/\hbar]J_r(\phi)$.

An environmental perturbation, such as external vibration, change of the external temperature, perturbation of the external electromagnetic fields, and so on, can induce a change of the internal structure of the Josephson junction. The values of $\omega_1(\phi)$ and $\omega_2(\phi)$ in Eq. (4) will vary when the internal structure of the Josephson junction changes. We describe the fluctuations caused by a change of the internal structure of the Josephson junction as stochastic external parameters $\omega_1(\phi)+\sigma_1\xi_1(\phi,t)$ and $\omega_2(\phi)+\sigma_2\xi_2(\phi,t)$ [8–13], in which $\xi_1(\phi,t)$ and $\xi_2(\phi,t)$ are the stochastic forces with respect to ϕ and t, and σ_1 and σ_2 are positive constants. The stochastic driving current is assumed to be only thermal noise $I(t)=\eta(t)$, which is Gaussian white noise with zero mean and correlation function $\langle\eta(t)\eta(t')\rangle=2D\delta(t-t')$. In this case, Eq. (4) becomes

$$\dot{\phi} = f(\phi) - \sigma_1\xi_1(\phi,t)\sin\frac{\phi}{2} - \sigma_2\xi_2(\phi,t)\sin(\phi+\phi_e) + \frac{eR(\phi)}{\hbar}\eta(t), \tag{5}$$

where $f(\phi)=-\omega_1(\phi)\sin(\phi/2)-\omega_2(\phi)\sin(\phi+\phi_e)$. In this three-superconducting-junction device, the multiplicative noises $\xi_1(\phi,t)$ and $\xi_2(\phi,t)$ and the additive noise $\eta(t)$ come from the external environmental perturbation and the thermal fluctuation, respectively. We assume that there are no correlations between the additive and multiplicative noises.

A. Gaussian noise model

In this section, we will show that the spatiotemporal noise has an effect on the transport of electron pairs. To do this, we should first give some assumptions. From the above analysis, we know that $\xi_k(\phi,t)$ is noise that is stochastic with respect to ϕ and t, and J_i and R have weak relations with ϕ. So we assume that $\xi_k(\phi,t)$ is Gaussian noise, $J_i(\phi)=J_i^{(0)}+(J_i^{(0)}/5)\exp(-\sin\phi)$ and $R(\phi)=R_0+(R_0/7)\exp(-\sin\phi)$ ($i=r,l$) [the assumptions about $J_i(\phi)$ and $R(\phi)$ are made only to show that the spatiotemporal noise has an effect on the transport of electron pairs]. In the dimensionless form, we set $(\hbar/e)=J_i^{(0)}=R_0=\sigma_k=1$ ($k=1,2$ and $i=r,l$). Then Eq. (5) becomes

$$\dot{\phi} = f(\phi) - \xi_1(\phi,t)\sin\frac{\phi}{2} - \xi_2(\phi,t)\sin(\phi+\phi_e) + R(\phi)\eta(t), \tag{6}$$

where

$$\begin{aligned} f(\phi) &= -\omega_1(\phi)\sin(\phi/2) - \omega_2(\phi)\sin(\phi+\phi_e) \\ &= -\sin(\phi/2)[1+(1/5+1/7)\exp(-\sin\phi)+(1/35) \\ &\quad\times\exp(-2\sin\phi)] - \sin(\phi+\phi_e)[1+(1/5+1/7) \\ &\quad\times\exp(-\sin\phi)+(1/35)\exp(-2\sin\phi)] \end{aligned}$$

and $R(\phi)=1+(1/7)\exp(-\sin\phi)$. In addition, $\xi_k(\phi,t)$ has zero mean and correlation functions $\langle\xi_k(\phi,t)\eta(t)\rangle_{ff}=0$ and $\langle\xi_k(\phi,t)\xi_{k'}(\phi',t')\rangle_{ff}=2D_k\delta_{kk'}w(\phi,\phi')\delta(t-t')$ with $w(\phi,\phi')=\frac{1}{2}[g(\phi,\phi')+g(\phi',\phi)]$ and $g(\phi,\phi')=\sin(\phi+\phi')+\frac{1}{2}\sin(2\phi+\phi')+2$, in which $\langle\,\rangle_{ff}$ denotes the average over the noise $\xi_k(\phi,t)$ ($k,k'=1,2$).

Using the formulas in Ref. [23] (see Ref. [24]), we can get the Fokker-Planck equation of Eq. (6) [24]. It is

$$\partial_t P(\phi,t) = -\partial_\phi A(\phi)P(\phi,t) + \partial_\phi^2 B(\phi)P(\phi,t), \tag{7}$$

where

$$\begin{aligned} A(\phi) &= f(\phi) + (D_1/4)\sin\phi\, w(\phi,\phi) + (D_2/2)\sin(2\phi \\ &\quad + 2\phi_e)w(\phi,\phi) + D_1\sin^2(\phi/2)\partial_\phi w + D_2\sin^2(\phi \\ &\quad + \phi_e)\partial_\phi w + DR'(\phi)R(\phi), \end{aligned}$$

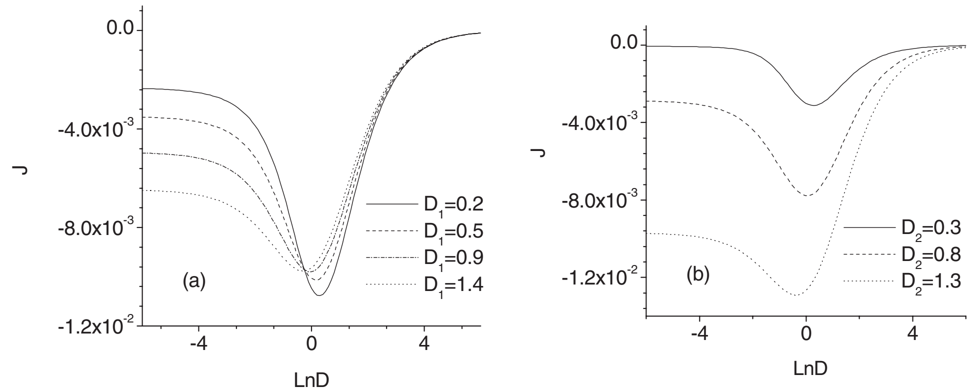

FIG. 2. Current J of electron pairs versus the natural logarithm of the additive noise strength D for the Gaussian noise model in the dimensionless form: (a) for different values of D_1 ($D_1=0.2$, 0.5, 0.9, and 1.4, respectively) with $D_2=1$; (b) for different values of D_2 ($D_2=0.3$, 0.8, and 1.3, respectively) with $D_1=1$.

and $B(\phi)=D_1\sin^2(\phi/2)w(\phi,\phi)+D_2\sin^2(\phi+\phi_e)w(\phi,hi)+DR^2(\phi)$ with $w(\phi,\phi)=\sin(2\phi)+(1/2)\sin(3\phi)+2$, $\partial_\phi w=\cos(2\phi)+(3/4)\cos(3\phi)$, and $R'(\phi)=-[(\cos\phi)/7]\times\exp(-\sin\phi)$.

Using the formulas in Ref. [5] (see Ref. [25]), in Figs. 2(a) and 2(b), we plot the current J [25] of the electron pairs versus the natural logarithm of the additive noise strength D in the dimensionless form [Fig. 2(a) corresponds to the current J versus the natural logarithm of the additive noise strength D for different values of D_1 ($D_1=0.2$, 0.5, 0.9, and 1.4, respectively) with $D_2=1$; Fig. 2(b) to the current J versus the natural logarithm of the additive noise strength D for different values of D_2 ($D_2=0.3$, 0.8, and 1.3, respectively)]. Some characteristic features can been seen from the figures: (1) the current of the electron pairs is a nonmonotonic function of the thermal noise strength D (or the temperature T, since the thermal noise strength is proportional to the temperature T); (2) the curves of the current of the electron pairs are always negative; (3) the curves for the current of the electron pairs as a function of D (or T) have wells, which is the reverse of the manifestation of the phenomenon of resonance; (4) with increase of D_1, the valley values of the curves for J versus the natural logarithm of the additive noise strength D ascend (and move toward the left), while with increase of D_2 the valleys of the curves for J versus the natural logarithm of the additive noise strength D descend (and also move to the left).

B. Dichotomous noise model

In this section, by another model, we will show again that the spatiotemporal noise has an effect on the transport of electron pairs. Of course, to do this, we should first give some assumptions as in Secs. II A. We assume that $\xi_1(\phi,t)$ and $\xi_2(\phi,t)$ in Eq. (6) are dichotomous noises. After replacing $\xi_1(\phi,t)$ and $\xi_2(\phi,t)$ by dichotomous noises $\zeta_1(\phi,t)$ and $\zeta_2(\phi,t)$ in Eq. (6), we have

$$\dot{\phi}=f(\phi)-\zeta_1(\phi,t)\sin\frac{\phi}{2}-\zeta_2(\phi,t)\sin(\phi+\phi_e)+R(\phi)\eta(t), \tag{8}$$

in which $f(\phi)$ and $R(\phi)$ are same as given in Eq. (6); the dichotomous noise $\zeta_1(\phi,t)$ has two values $v_1(\phi)$ and b_1, and the transition rate of $\zeta_1(\phi,t)$ from $v_1(\phi)$ to b_1 or vice versa is γ_1; the dichotomous noise $\zeta_2(\phi,t)$ has two values $v_2(\phi)$ and b_2, and the transition rate of $\zeta_2(\phi,t)$ from $v_2(\phi)$ to b_2 or vice versa is γ_2.

In order to make the calculation conveniently, we let $\zeta_k(\phi,t)=\varepsilon_k(\phi,t)+g_k(\phi)$ ($k=1,2$) where $\varepsilon_k(\phi,t)$ is a dichotomous noise, which takes two function values $c_k(\phi)$ and $-c_k(\phi)$, $g_k(\phi)$ is a function of ϕ, and the transition from $c_k(\phi)$ to $-c_k(\phi)$ or vice versa is γ_k. Using the relations between the dichotomous noise $\zeta_k(\phi,t)$ and the dichotomous noise $\varepsilon_k(\phi,t)$, we can get $c_k(\phi)=[v_k(\phi)-b_k]/2$ and $g_k(\phi)=[v_k(\phi)+b_k]/2$. Substituting $\zeta_k(\phi,t)=\varepsilon_k(\phi,t)+g_k(\phi)$ ($k=1,2$) into Eq. (8), we get

$$\dot{\phi}=w(\phi)-\varepsilon_1(\phi,t)\sin\frac{\phi}{2}-\varepsilon_2(\phi,t)\sin(\phi+\phi_e)+R(\phi)\eta(t), \tag{9}$$

where $w(\phi)=f(\phi)-g_1(\phi)\sin(\phi/2)-g_2(\phi)\sin(\phi+\phi_e)$.

For simplicity, below we assume that $v_1(\phi)=v_2(\phi)=v(\phi)$, $b_1=b_2=b$, and $\gamma_1=\gamma_2=\gamma$. So we have $\zeta_1(\phi,t)=\zeta_2(\phi,t)$ [we assume $\zeta_1(\phi,t)=\zeta_2(\phi,t)=\zeta(\phi,t)$]. In this case, we assume $c_1(\phi)=c_2(\phi)=c(\phi)$, $g_1(\phi)=g_2(\phi)=g(\phi)$, and $\varepsilon_1(\phi,t)=\varepsilon_2(\phi,t)=\varepsilon(\phi,t)$. Now Eq. (9) becomes

$$\dot{\phi}=w(\phi)+w_1(\phi)\varepsilon(\phi,t)+R(\phi)\eta(t), \tag{10}$$

in which $w(\phi)=f(\phi)+g(\phi)w_1(\phi)$ and $w_1(\phi)=-\sin(\phi/2)-\sin(\phi+\phi_e)$.

The master equation of Eq. (10) is [14]

$$\partial_t P_1=-\partial_\phi[w(\phi)+w_1(\phi)c(\phi)]P_1+\gamma P_2-\gamma P_1+D\partial_\phi R(\phi)\partial_\phi R(\phi)P_1, \tag{11}$$

$$\partial_t P_2=-\partial_\phi[w(\phi)-w_1(\phi)c(\phi)]P_2+\gamma P_1-\gamma P_2+D\partial_\phi R(\phi)\partial_\phi R(\phi)P_2, \tag{12}$$

where $P_1=P(\phi,t,c(\phi))$ and $P_2=P(\phi,t,-c(\phi))$. Let $P=P_1+P_2$ and $P'=P_1-P_2$, from Eqs. (11) and (12); we get

$$\partial_t P(\phi,t) = -\partial_\phi w(\phi)P(\phi,t) - \partial_\phi w_1(\phi)c(\phi)P'(\phi,t) + D\partial_\phi R(\phi)\partial_\phi R(\phi)P(\phi,t), \quad (13)$$

$$\partial_t P'(\phi,t) = -\partial_\phi w(\phi)P'(\phi,t) - \partial_\phi w_1(\phi)c(\phi)P(\phi,t) - 2\gamma P'(\phi,t) + D\partial_\phi R(\phi)\partial_\phi R(\phi)P'(\phi,t). \quad (14)$$

The formal solution of Eq. (14) is

$$P'(\phi,t) = \int_0^t e^{-\hat{A}(t-\tau)}[-\partial_\phi w_1(\phi)c(\phi)P(\phi,t)]d\tau, \quad (15)$$

in which $\hat{A}=2\gamma+\partial_\phi w(\phi)-D\partial_\phi R(\phi)\partial_\phi R(\phi)$. Substituting Eq. (15) into Eq. (13), we obtain the probability density equation

$$\partial_t P(\phi,t) = -\partial_\phi w(\phi)P(\phi,t) + D\partial_\phi R(\phi)\partial_\phi R(\phi)P(\phi,t) - \partial_\phi w_1(\phi)c(\phi)\int_0^t e^{-\hat{A}(t-\tau)} \times[-\partial_\phi w_1(\phi)c(\phi)P(\phi,\tau)]d\tau. \quad (16)$$

Here $R(\phi)$ and $w(\phi)$ are periodic functions of ϕ. So, in the case $2\gamma \gg D|\max[R(\phi)]|$ and $|\max[w(\phi)]|$, the integral on the right-hand side of Eq. (16) stems mainly from the values near $t=\tau$, so we can approximately take $P(\phi,\tau)\simeq P(\phi,t)$. Then Eq. (16) becomes

$$\partial_t P(\phi,t) = -\partial_\phi w(\phi)P(\phi,t) + D\partial_\phi R(\phi)\partial_\phi R(\phi)P(\phi,t) + \partial_\phi w_1(\phi)c(\phi)\hat{A}^{-1}\partial_\phi w_1(\phi)c(\phi)P(\phi,t). \quad (17)$$

Taking into account the condition $2\gamma \gg D|\max[R(\phi)]|$ and $|\max[w(\phi)]|$, we can obtain the approximate Fokker-Planck equation

$$\partial_t P(\phi,t) = -\partial_\phi A(\phi)P(\phi,t) + \partial_\phi^2 B(\phi)P(\phi,t), \quad (18)$$

where

$$A(\phi) = w(\phi) + D[\partial_\phi R(\phi)]R(\phi) + (1/2\gamma) \times\{\partial_\phi[w_1(\phi)c(\phi)]\}w_1(\phi)c(\phi)$$

and $B(\phi)=DR^2(\phi)+[1/(2\gamma)]w_1^2(\phi)c^2(\phi)$.

Using the formulas in Ref. [5] (see Ref. [25]), in Fig. 3, we plot the current J [25] versus the natural logarithm of the thermal noise strength D with $\gamma=1000$, $b=-1$, and $v(\phi)=2+\sin\phi$ in the dimensionless form. From the figure, we can see that the current is negative and exhibits a well with increasing D (or the temperature T).

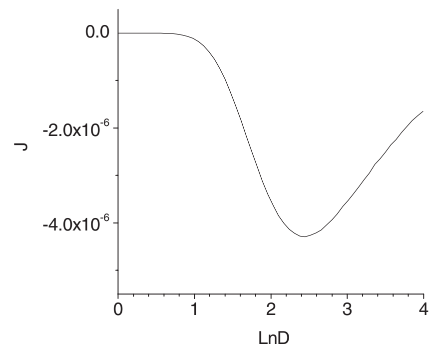

FIG. 3. Current J of electron pairs versus the natural logarithm of the additive noise strength D for the dichotomous noise model in the dimensionless form, with $\gamma=1000$, $b=-1$, and $v(\phi)=2+\sin\phi$.

III. CONCLUSION AND DISCUSSION

In conclusion, we have shown that, in superconducting junctions, spatiotemporal noise can appear and induce nonzero current due to symmetry breaking. The results for the nonzero current of electron pairs caused by the spatiotemporal noise in this paper are only mathematically and physically theoretical; it remains yet to be verified by experiments whether spatiotemporal noise and the transport of electron pairs exist for superconducting junctions. Our results can provide a theoretical foundation for the further investigation of superconducting junctions, especially in experiments.

It is necessary to give some explanation of the origin of the nonzero probability current of electron pairs. Consider a solution $\phi(t)$ of Eq. (5) for a given realization of the noise. Then $-\phi(t)$ is also a solution of Eq. (5), with t replaced by $-t$. If $\xi_1(\phi,t)$ and $\xi_2(\phi,t)$ are not related to ϕ or are stochastic only with respect to time, this solution $-\phi(t)$ will have the same probability as $\phi(t)$, and there will be no symmetry breaking. However, in the presence of noise correlation with ϕ, the probability does not have this symmetry. So a nonzero flux of electron pairs can appear due to the symmetry breaking.

In the two models above, we find that the nonzero current caused by spatiotemporal noise is always negative. From the calculated results, one may manipulate the noise to reduce the current to the lowest degree. In the case of a given environmental perturbation, we may appropriately adjust the temperature to make the current depart from the well value by taking into account that in Eqs. (6) and (8) the internal thermal noise strength D is proportional to the temperature T. For a given temperature, we should adopt measures to reduce and avoid environmental perturbations in order to make the absolute value of the current as low as possible.

ACKNOWLEDGMENTS

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[22] For the superconducting junction device in Fig. 1, in the overdamped case, if we take $R_1=R_1=R_2=R/2=R_r/2$, $J_1=J_2=J_l$, and $\phi=\phi_l$, we have the phase differential equation $(\hbar/(eR))\dot{\phi}=-J_l\sin(\phi/2)-J_r\sin(\phi+\phi_e)+I(t)$ where ϕ_e satisfies the phase around the loop: $\phi_l-\phi_r=-\phi_e+2\pi n$ with $\phi_e=2\pi\Phi_e/\Phi_0$ in which $\Phi_0=\hbar/(2e)$ and Φ_e is the external magnetic flux [7].

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[24] If the Langevin equations of a system satisfy $\dot{x}_i=f_i(\vec{x},t)+\Sigma_j g_{ij}(\vec{x},t)\xi_{ij}(\vec{x},t)$, with $\langle\xi_{ij}(\vec{x},t)\rangle=0$, $\langle\xi_{ij}(\vec{x},t)\xi_{kl}(\vec{y},s)\rangle=2\delta_{jl}w_{ik}^{j}(\vec{x},\vec{y},t)\delta(t-s)$, where $\vec{x}=\{x_1,x_2,\ldots,x_N\}$, $f_i(\vec{x},t)$ and $g_{ij}(\vec{x},t)$ are arbitrary deterministic functions of variables $\vec{x}$ and t, $\xi_{ij}(\vec{x},t)$ is spatiotemporal Gaussian white noise, $x_i=x_i(t)$, $y_i=x_i(s)$, $\langle\,\rangle$ denotes averaging over spatiotemporal noise, and $w_{ik}^{j}(\vec{x},\vec{y},t)$ are deterministic functions of $\vec{x}$, $\vec{y}$ and t $[w_{ik}^{j}(\vec{x},\vec{y},t)>0]$, the Fokker-Planck equation of the system is [23] $\partial_t P(\vec{x},t)=-\Sigma_i^N\partial_{x_i}D_i^{(1)}(\vec{x},t)P(\vec{x},t)+\Sigma_{kl}^N\partial_{x_k}\partial_{x_l}D_{kl}^{(2)}(\vec{x},t)P(\vec{x},t)$ with $D_i^{(1)}(\vec{x},t)=f_i(\vec{x},t)+\Sigma_l^N\Sigma_j\{(\partial_{x_l}w_{li}^{j})g_{ij}(\vec{x},t)g_{lj}(\vec{x},t)+[\partial_{x_l}g_{ij}(\vec{x},t)]g_{lj}(\vec{x},t)w_{li}^{j}(\vec{x},\vec{x},t)\}$ and $D_{kl}^{(2)}(\vec{x},t)=\Sigma_j g_{lj}(\vec{x},t)g_{kj}(\vec{x},t)w_{kl}^{j}(\vec{x},\vec{x},t)$ in which $\partial_{x_l}w_{li}^{j}=\partial_{x_l}w_{li}^{j}(\vec{x},\vec{y},t)|_{\vec{y}=\vec{x}}$.

[25] If a spatially periodic system (whose one period is $L=a-b$) has the Fokker-Planck equation: $\partial_t P(x,t)=-\partial_x A(x)P(x,t)+\partial_x^2 B(x)P(x,t)$, the stationary probability current J satisfies [5] $J=N\{1-\exp[-\phi(b)]\}$ where N is the normalization constant for the stationary probability distribution, $\phi(x)$ is defined as $\phi(x)=\int_a^x[A(x')/B(x')]dx'$.